

Axion phantom energy

Pedro F. González-Díaz

*Centro de Física "Miguel A. Catalán," Instituto de Matemáticas y Física Fundamental,
Consejo Superior de Investigaciones Científicas, Serrano 121, 28006 Madrid, Spain*

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The existence of phantom energy in a universe which evolves to eventually show a big rip doomsday is a possibility which is not excluded by present observational constraints. In this paper it is argued that the field theory associated with a simple quintessence model is compatible with a field definition that is interpretable in terms of a rank-3 axionic tensor field, whenever we consider a perfect-fluid equation of state that corresponds to the phantom energy regime. Explicit expressions for the axionic field and its potential, both in terms of an imaginary scalar field, are derived, which show that these quantities both diverge at the big rip, and that the onset of phantom-energy dominance must take place just at present.

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INTRODUCTION

Cosmology has become the current common place where the greatest problems of physics are being concentrated or have most clearly manifested themselves. Along with the nature of dark matter, the origin of supermassive black holes and several unexplained huge amounts of released astrophysical energy are only some illustrative examples. However, perhaps the biggest problem of all in physics is the so-called dark energy problem [1] in which it is known that nearly 70% of the total energy of the Universe is in the form of an unobserved vacuum energy which is responsible for the present accelerating expansion of the Universe [2,3] and whose nature remains a mystery [4]. There has been a rather large influx of papers in recent years trying to shed some light on the existence and kind of possible stuff that may make up dark energy [5].

Four main candidates to represent dark energy have been hitherto suggested: A positive cosmological constant [6], the quintessence fields (which may [7] or may not [8] be tracked), some generalizations of the Chaplygin gas [9], and the so-called tachyon model of Padmanabhan *et al.* [10]. Even though such models could all be accommodated by present observational constraints, these models pose new or traditional problems in such a way that none of them becomes completely satisfactory. In particular, cosmological data from current observations do not exclude [11], but may suggest [12], values of the parameter ω in the perfect-fluid equation of state $p = \omega\rho$ of the most popular quintessence models which are smaller than -1 . If this were the case, then dark energy would become what is now named phantom energy [13], an entity violating the dominant energy condition and thereby allowing the natural occurrence of wormholes and ringholes and even their corresponding time machines in the universe [14].

Caldwell, Kamionkowski, and Weinberg have recently noticed [15] that in the framework of quintessence models phantom energy may lead to a doomsday for the universe—which would take place at a big rip singularity—once clusters, galaxies, stars, planets, and, ultimately, nucleons and leptons in it are all ripped apart. Even though such a big rip does not take place in some phantom-energy models that use

Chaplygin-like equations of state [16], it seems to be the simplest and most natural possibility stemming from phantom energy. On the other hand, the big rip cosmic scenario interestingly adds an extra qualitative feature to the known set of cosmological models as it introduces a curvature singularity, other than that at the big bang, at a finite, nonzero value of the cosmic time. However, in spite of the feature that the blowing up of the scale factor appears to be unavoidable in models with equation of state $p = \omega\rho$ and $\omega < -1$, a complete account of the nature and properties of the field theory associated with such models has not been done yet. This paper aims at investigating the characteristics of the massless scalar field which allow emergence of a big rip singularity in the simplest of such phantom-energy models, assuming a perfect-fluid equation of state. It will be shown that the stuff making up phantom energy can be interpreted to be a vacuum sea of cosmic axions, which can be described in terms of the kind of rank-3 tensor field strengths predicted in supergravity and string theories.

This paper can be outlined as follows. In Sec. II we argue in favor of the idea that superlight axions are the source of cosmic phantom energy. A simple cosmological model accounting for a big rip singularity in the case that the vacuum is filled with phantom energy is discussed in Sec. III. Section IV contains the solution of the phantom field theory within the simple cosmological model of Sec. III. We check that both the scalar field and its potential also have a singularity at the big rip. The results are summarized in Sec. V.

AXIONS AS THE SOURCE OF PHANTOM ENERGY

The definition of the massless scalar field ϕ which is assumed to make up dark energy is usually taken to be the conventional simplest one; that is, in terms of pressure p and energy density ρ ,

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad (1)$$

where $V(\phi)$ is the field potential. From the equation of state $p = \omega\rho$ and Eq. (1) it immediately follows that

$$\rho = \frac{\dot{\phi}^2}{1 + \omega}. \quad (2)$$

Thus, if the weak energy condition $\rho \geq 0$ is taken to be always satisfied [17], the requirement $p + \rho < 0$, $\omega < -1$ from phantom energy in this kind of model necessarily implies that the field ϕ ought to be purely imaginary; that is to say, if the vacuum energy density for a phantom vacuum as referred to any timelike observer has to be positive, then the massless scalar field making up the phantom stuff should be purely imaginary. I will argue in what follows that, in the classical framework, such a massless, pure imaginary scalar field actually represents an axion describable as a rank-3 antisymmetric tensor field, considering later the associated cosmic theory for such a field. In fact, the Lorentzian action that couples such an axion field to gravity can generally be written as

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} - A^2 + L_m \right), \quad (3)$$

where L_m is the Lagrangian for observable matter and $A^2 \equiv A_{\mu\nu\alpha} A^{\mu\nu\alpha}$ if we choose the axion to be given by a three-form $A = dB$ field strength so that $dA = 0$, that is, as a rank-3 antisymmetric tensor field strength of a type arising in supergravity-theory motivated quantum-gravity solutions [18]. The equations of motion derived from action (3) are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 16\pi G \left(3A_{\mu\nu}^2 - \frac{1}{2} g_{\mu\nu} A^2 + T_{\mu\nu}^{(m)} \right), \quad (4)$$

$$d^*A = 0, \quad (5)$$

in which $A_{\mu\nu}^2 = A_{\mu\alpha\beta} A_{\nu}^{\alpha\beta}$, the asterisk denotes the Hodge dual, and $T_{\mu\nu}^{(m)}$ is the momentum-energy tensor for ordinary observable matter. One can now check that any explicit solution to these equations of motion subject to the usual Friedmann-Robertson-Walker (FRW) symmetry for the metric and the corresponding spherically symmetric ansatz for the axion antisymmetric tensor $A = f(r)\epsilon$ [with ϵ the volume form which, when integrated over a surface of constant radius, yields the area of the unit three-sphere $A = 2\pi^2/\Gamma(2)$], if we set, e.g., $L_m = 0$ and $f(r) = \text{const}$, is the same as the solution obtained from the equations of motion derived from the FRW action integral describing coupling of a real massless scalar field ϕ to gravity where an extra boundary term accounting for the axion properties, $-a^3 \dot{\phi} \phi|_0^T$ (a being the scale factor), is added; i.e., in the flat geometry case

$$S' = \frac{1}{16\pi G} \int_0^T dt a^3 \left[-\frac{\dot{a}^2}{N} + 8\pi G \frac{\dot{\phi}^2}{N} \right] - \frac{a^3 \dot{\phi} \phi}{N} \Big|_0^T, \quad (6)$$

where N is the lapse function. The key point then is that the solution to the above equations of motion is even preserved when we omit such an extra boundary term in the action for the scalar field ϕ , provided this scalar field is simultaneously rotated to the pure imaginary axis, $\phi \rightarrow i\Phi$. If the scalar field

is equipped with a potential $V(\phi)$, then an extra term $-\int_0^T dt N A^3 V(\phi)$ should be added to the action. In that case, the axionic action can be obtained by simply rotating $\phi \rightarrow i\Phi$ in an action containing the above potential term, without any extra boundary term.

Such a property, which was first noticed in Euclideanized solutions such as wormholes and other instantons describing nucleation of baby universes [19], is actually independent of the metric signature and, therefore, applies also to our Lorentzian cosmological context. Thus, one can interpret that the stuff that makes up phantom energy can be regarded as a rank-3 antisymmetric tensor axionic field. It is usually thought that axions can explain the absence of an electrical dipole moment for the neutron and thereby solve the so-called strong *CP* problem [20]. The axions are chargeless and spinless particles with very tiny mass which interact with ordinary matter only very weakly. Such particles are believed to have been abundantly produced in the big bang. It is worth noticing that, whereas relic axions are an excellent candidate for the dark matter in the universe [21], their vacuum quantum background could make cosmic phantom energy.

The coincidence and fine-tuning problems could be thought to become exacerbated in the present scenario where one sets ω constant and < -1 . However, in dark energy models such as the generalized Chaplygin gas [9] and tachyon models [10], dark energy and dark matter are described as separate limiting cases from an existing unique field. Partly inspired by these models, one could naively assume the existence of a unique axion field which, when excited, would make dark matter, and when at its vacuum ground state would be the source of phantom energy. Coincidence time could then be interpreted as the time when both the vacuum and the excited states are approximately equally populated. Of course, this would not solve the coincidence and fine-tuning problems but provided some explanation of these problems and of the prediction of generating such strangely small amounts of homogeneously distributed axions. It remains nevertheless an intriguing possibility that such a small value of the cosmological constant can be supplied by the potential energy density of an “ultrainvisible” axion field, which can be dubbed the quintessence axion [22].

THE COSMOLOGICAL MODEL

We shall consider in what follows a simple model where the massless scalar field ϕ is a quintessential field, equipped with a potential $V(\phi)$, which is minimally coupled to Hilbert-Einstein gravity. The action for this model is

$$S_c = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} + \frac{1}{2} \partial^i \phi \partial_i \phi - V(\phi) + L_m \right), \quad (7)$$

where L_m is again the Lagrangian for observable matter fields. If we again restrict ourselves to the case where (i) $L_m = 0$, (ii) both the scalar field and Hilbert-Einstein gravity satisfy the Friedmann-Robertson-Walker symmetry, and (iii) we assume a perfect fluid equation of state, then by integrating the conservation law for cosmic energy, $d\rho = -3(p$

$+p)da/a$, it turns out that the energy density for the quintessence scalar field will be given by

$$\rho = Ra^{-3(1+\omega)}, \quad (8)$$

in which R is an integration constant. From the Friedmann equation derived from the action (7) subject to the FRW symmetry and $L_m=0$,

$$\left(\frac{\dot{a}}{a}\right)^2 = Aa^{-3(1+\omega)}, \quad A = \frac{8\pi GR}{3}, \quad (9)$$

we can obtain the solution

$$a(t) = \left[a_0^{3(1+\omega)/2} + \frac{3(1+\omega)\sqrt{A}}{2}(t-t_0) \right]^{2/3(1+\omega)}, \quad (10)$$

where a_0 and t_0 are the initial radius and time, respectively. Note that for $\omega > -1$ this solution describes an accelerating universe whose scale factor increases toward infinity as $t \rightarrow \infty$. We are here most interested in the case, not yet observationally excluded, where $\omega < -1$, which corresponds to the so-called phantom dark energy for which the dominant energy condition is generally violated, i.e. [15],

$$p + \rho < 0, \quad (11)$$

even though the energy density is surprisingly ever increasing. Notice, furthermore, that in this case the scale factor blows up at a finite time,

$$t_* = t_0 + \frac{2}{3(|\omega| - 1)a_0^{3(|\omega|-1)/2}}, \quad (12)$$

giving rise to a “big rip” [15]. One would expect that in the vicinity of time t_* the size of all local geometrical objects increased in a rather dramatic way. We see that the larger a_0 and $|\omega|$ the nearer the doomsday and hence the time expected to have macroscopically increased geometrical objects in our universe.

THE FIELD THEORY

Now we shall consider the field theory that is associated with phantom energy, starting with the general formalism for any value of ω . From the equation of motion for the scalar field ϕ ,

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV(\phi)}{d\phi}, \quad (13)$$

where $H \equiv \dot{a}/a$ and $V(\phi)$ again is the potential for the field ϕ , and the expression for kinetic energy of the field [Eq. (2)], we can obtain a relation between $dV/d\phi$ and the scale factor, which can be expressed in the form

$$\frac{3}{2}\sqrt{A(1+\omega)}(1-\omega)a^{-3(1+\omega)} = -\frac{dV}{d\phi}. \quad (14)$$

On the other hand, integrating the equation of motion for the field ϕ we can derive the dependence of this field with time t ,

$$\phi(t) = \frac{2}{3\sqrt{1+\omega}\ell_p} \ln \left[\frac{a_0^{3(1+\omega)/2} + [3(1+\omega)\sqrt{A}/2](t-t_0)}{B_0} \right], \quad (15)$$

where $\ell_p = \sqrt{R/A}$ is the Planck length and B_0 is an integration constant.

From Eqs. (10) and (15) an expression for the scale factor in terms of the scalar field can now be derived. It is

$$a(\phi) = B_0^{2/3(1+\omega)} \exp \left(\frac{\ell_p \phi}{\sqrt{1+\omega}} \right). \quad (16)$$

Using Eqs. (14) and (16) we finally obtain the expression for the field potential for any $\omega < -1/3$ as a function of the field ϕ :

$$V(\phi) = V_0 + \frac{1}{2}(1-\omega)\sqrt{R}B_0^{-2} \exp(-3\sqrt{1+\omega}\ell_p\phi), \quad (17)$$

in which V_0 is a constant. Potentials with this form naturally arise in supergravity models [23] and have been used in a variety of context, ranging from accelerating expansion models [24] to cosmological scaling solutions [25].

Together with the expression for the field ϕ in terms of time t , this expression solves the field-theory problem for any value of $\omega < -1/3$. According to our discussion above, for axionic phantom energy we have to perform the continuation $\phi \rightarrow i\Phi$ in addition to taking $\omega < -1$. In such a case, the solution of the field theory problem would read

$$\Phi(t) = -\frac{2}{3\sqrt{|\omega|-1}\ell_p} \times \ln \left[\frac{a_0^{-3(|\omega|-1)/2} - [3(|\omega|-1)\sqrt{A}/2](t-t_0)}{B_0} \right], \quad (18)$$

$$V(\Phi) = V_0 + \frac{1}{2}(1+|\omega|)\sqrt{R}B_0^{-2} \exp(3\sqrt{|\omega|-1}\ell_p\Phi). \quad (19)$$

As to the physical motivation for the exponential axion potential (19), I will briefly comment on the influence that the matter-field sources may have on the stability of the present axion model of phantom energy. If, corresponding to the physically interesting case that $V_m \neq 0$, we let the influence of matter sources on the expansion factor immediately before phantom domination have the form of a perturbation, and analyze the resulting model via phase space [26], it can be seen that if we choose $V_0=0$ then potential (19) and the solution given by Eq. (10) (with $\omega < -1$) and Eq. (18), that is, for

$$t = t_0 + \frac{2[1 - (a_0/a)^{3(|\omega|-1)/2}]}{3a_0^{3(|\omega|-1)/2}(|\omega|-1)\sqrt{A}},$$

correspond (see, e.g., Ref. [26]) to a dynamical attractor for phantom-energy domination and a catastrophic big rip with $\omega < -1$ and

$$\Gamma = \frac{V(\Phi)' V(\Phi)''}{[V(\Phi)']^2} = 1,$$

at a critical point defined by

$$8\pi G\lambda_c = -\frac{V(\Phi)'}{V(\Phi)} = -3\sqrt{|\omega|-1}\ell_p,$$

for which $\omega = -1 - \lambda_c^2/3$. (See also phantom models with Born-Infeld type Lagrangians [27].)

Note that the potential $V(\Phi)$ takes on the value

$$V = V_0 + \frac{1}{2}(1 + |\omega|)\sqrt{R}a_0^{3(|\omega|-1)} \quad (20)$$

at the initial time $t = t_0$ to steadily increase thereafter up to infinity when $t = t_*$, at the big rip. From that moment on the potential would continuously decrease, tending to reach its constant minimum value V_0 as $t \rightarrow \infty$ (see solid curves in Fig. 1). That behavior is in sharp contrast to that of the potential $V(\phi)$ for dark energy with $\omega > -1$, which starts with the value

$$V = V_0 + \frac{1}{2}(1 - \omega)\sqrt{R}a_0^{-3(1+\omega)}$$

and monotonically decreases down to the value V_0 as $t \rightarrow \infty$, as is shown by the dashed curve in Fig. 1. We have solved in this way the field theory which is associated with dark and phantom energy if the latter is interpreted as originating from the existence of an axionic field which is expressible as a massless, purely imaginary scalar field. The explicit emergence of axions in the cosmological context of phantom dark energy appears to be just another example where such rather elusive particles are invoked as candidates for inexorably needed cosmic constituents which are themselves defined to be unobservable. Actually, apart from being a key ingredient to solve the strong CP problem [20] or to represent quantum-gravity topology changes [19], axions have already been widely used to solve a variety of astrophysical and cosmological shortcomings [28].

It appears of some interest to briefly comment next on a potentially fruitful consequence of the connection of axions with our cosmological model. If we are actually living in a universe with phantom energy where a big rip singularity will occur which can never be circumvented by causally violating connections to the future, then the contracting branch of solution (9) can be physically disregarded as the big rip singularity is a true curvature singularity. In such a case, only the branch of potential $V(\Phi)$ before the singularity can have physical significance, and hence the global minimum of that

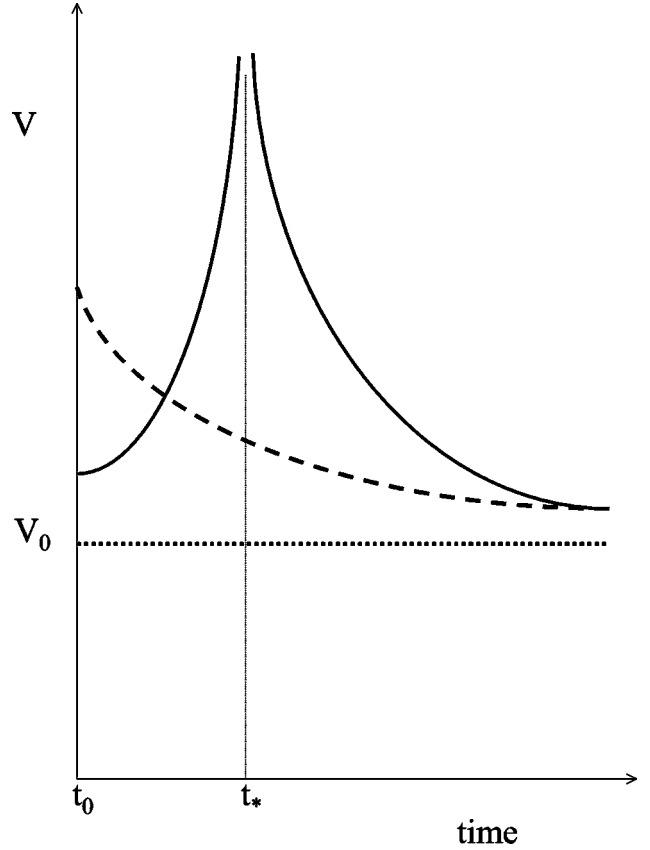


FIG. 1. The potential V for imaginary (solid lines) and real (dashed line) scalar field. The scales for the potential and time are arbitrary. t_* is the time at which the big rip singularity would occur in the future. It is worth noticing that the singularity in the space-time curvature at t_* coincides with a singularity in the imaginary field Φ and the potential $V(\Phi)$. In constructing this figure it has been assumed that the initial scale factor and the parameter entering the respective equations of state are such that $a_0 \gg [(1 - \omega_d)/(1 + |\omega_p|)]^{1/[3(\omega_d + |\omega_p|)]}$, where $\omega_d > -1$ and $\omega_p < -1$, with a_0 sufficiently large.

potential would be at $t = t_0$ with the value given by Eq. (20). It corresponds to a value of the field

$$\Phi_m = \frac{2}{3\ell_p\sqrt{|\omega|-1}} \ln(B_0 a_0^{3(|\omega|-1)/2}). \quad (21)$$

Interpreting the potential (19) as that axion potential resulting from the QCD nonperturbative effects, we can take [29] $\Phi_m = f_A \theta_{\text{eff}}$, with f_A a constant and θ_{eff} the effective θ parameter resulting after the diagonalization of the quark masses in the QCD Lagrangian. Setting then $f_A = 2/[3\ell_p\sqrt{|\omega|-1}]$, $\theta_{\text{eff}} = \ln(B_0 a_0^{3(|\omega|-1)/2})$, and $B_0 = 1$ it follows that, since $\theta_{\text{eff}} < 10^{-9}$ [29], the initial radius of the phantom-energy universe a_0 should be very close to unity, that is, the onset of such a phantom-energy regime, if it ever occurred at all, must always be placed at nearly the present epoch. Actually, if θ_{eff} is canceled to completely solve the strong CP problem [29], then the choice $B_0 = 1$ implies that a_0 would exactly satisfy $a_0 = 1$. In this case, the observa-

tional setting of the value of the state equation parameter ω would also set the value of a_0 , and hence of time t_* .

A potential problem with the proposal in this paper could be that, whereas current axion theories have a cutoff on the scale of inflationary energy, typically at around the grand unified theory characteristic energy of 10^{16} GeV [29], in order to avoid phantom energy decaying into gravitons, a cutoff of less than 100 MeV should be introduced in the effective phantom theory [30]. Notwithstanding, while the former cutoff appears relevant for dark matter axions which can initially be highly excited, the axion field proposed in this paper as the source of phantom energy can never be excited outside the vacuum. On the other hand, if we keep the current values for color anomaly of the Peccei-Quinn symmetry, the pion and quark masses and the pion decay constant, then the above Peccei-Quinn symmetry-breaking scale, $f_A \sim 10^{19}(|\omega| - 1)^{-1}$ GeV, would imply a mass of the phantom axions of

$$m_A \sim 0.62 \text{ eV} \frac{10^7 \text{ GeV}}{f_A} \sim 10^{-12} \sqrt{|\omega| - 1} \text{ eV},$$

i.e., just at the extremal minimum of the allowed values for

the axion mass, and clearly well below the cutoff required by Carroll *et al.* [30] for phantom energy to be stable.

SUMMARY

Imposing the weak energy condition, the negative kinetic energy for a phantom field is interpreted as originating from superlight axions. We have built up a simple cosmological model encompassing constant equations of state for both $\omega > -1$ and $\omega < -1$. In the latter case the universe evolves toward a singularity at finite time. The field theory is then solved for all cases, deriving an increasing exponential potential for the phantom field which also diverges at the singularity and shows a dynamical attractor also when matter fields are present that inexorably leads to a catastrophic big rip.

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